LEEDS PHILOSOPHICAL
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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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SIR R. S. BALL, LL.D., F.R.S., PRESIDENT, in the Chair

William James Stewart Lockyer, M.A., Ph.D., 16 Penywern Road, South Kensington, S.W.,

was balloted for and duly elected a Fellow of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Samuel Cooke, M.A., Principal of the College of Science, Poona, India, 2 Mortimer Road, Clifton, Bristol (proposed by Sir R. S. Ball) ;

Thomas Charlton Hudson, B.A., Assistant, Nautical Almanac Office, 3 Verulam Buildings, Gray's Inn, W.C. (proposed by A. M. W. Downing) ;

David Hunter, John O'Groat's Villa, Cambuslang, Glasgow (proposed by John Dansken) ;

Alfred E. Larkman, Navigation Teacher, 56 Colville Road, Leytonstone, E. (proposed by W. E. Plummer) ;

Edmund Taylor Whittaker, B.A., Fellow of Trinity College, Cambridge (proposed by Sir R. S. Ball) ;

Walter Wickham, First Assistant, Radcliffe Observatory, Oxford (proposed by A. A. Rambaut).

Eighty presents were announced as having been received since the last meeting, including, amongst others :—

A $3\frac{1}{4}$-inch Equatorial Telescope, by Ross, with tripod stand, micrometer, &c., presented by Mrs. Mann ; W. F. Denning, The Great Meteoric Shower of November ; C. E. Peek, Variable Star Notes, No. 2 ; G. Johnstone Stoney, On Atmospheres upon Planets and Satellites, presented by the Authors ; Bonn Observatory, Eigenbewegungen von 335 Sternen, von F. Küstner ; Lick Observatory, Photographic Atlas of the Moon, plates 6–19, presented by the Observatories ; Photographs of nebulæ made with 24-inch reflector (lantern slides), presented by Mr. W. E. Wilson.

A determination of the Latitude-Variation and of the Constant of Aberration, from observations made at the Royal Observatory, Cape of Good Hope, 1892–94. By W. H. Finlay, M.A.

In the early part of 1892 a series of observations with the zenith telescope was commenced by Dr. Gill, for the purpose of determining the latitude-variation at the Cape, and the constant of aberration ; but pressure of other work led him to transfer the investigation to me at the end of the year.

Observations were continued throughout 1893 and 1894, but no use has been made of those taken after 1894 April. At this date the ether in the level suddenly evaporated to such an extent as to render work impossible. An attempt was made to refill the tube on the spot and observations were continued, but the result was not satisfactory : the bubble became very sluggish and sticky in places, so that the observations after this operation are affected with large accidental errors, and are not nearly comparable in accuracy with the earlier ones. Indeed, it may be said that the level was a weak point all through.

The value of one revolution of the micrometer-screw was determined from a number of transits of close polar stars at elongation, and the screw errors were carefully investigated.

Three groups of stars, each consisting of three pairs, were selected at R.A. 7^h, 15^h, 23^h respectively, and another three at R.A. 3^h, 10^h, 19^h. The programme laid down was that eight observations of one of these groups should be made soon after sunset, and eight of another before sunrise, as nearly as possible on the same dates. Care was also taken to secure as many sets starting "lamp east" as starting "lamp west." The total number of pairs observed was 621.

The reductions to apparent place were interpolated for the time of transit, and the small terms depending on the Moon's longitude were taken into account. The proper motions were

adopted from a discussion of all the available declinations of the stars, and in some cases they proved to be considerable—*e.g.* Lac. 2437, $-''\cdot334$: Lac. 2608, $+''\cdot370$: L^1 Puppis, $-''\cdot118$: Lac. 8109, $-''\cdot16$.

From every observation of a pair a correction to the adopted latitude was obtained, and the means of about four nights were taken for investigation. These were all considered to have equal weight, as no observations of a group were included unless at least two pairs were observed.

For a general solution the equations of condition were put in the form

$$C + x \cos M + y \sin M + x' \cos N + y' \sin N - A\rho = n,$$

where C is a constant for each group and depends on the assumed declinations of the stars and the adopted mean latitude : x, y, x', y' are coefficients of the periodic variation of latitude.

M and N are angles which go through their periods uniformly in 427 days and 365 days respectively, and are reckoned from 1893·0.

A is the mean of the aberration terms in the stars' reductions to apparent place, divided by 20·445.

ρ is the correction to the constant of aberration ($20''\cdot445$).

n is the mean of the observed corrections to the assumed latitude.

It was not anticipated that there would be any sensible personal equation between the two observers, and no particular care was taken to secure a comparison between them. Both observers took part in the observations of groups 4 and 5 at the end of 1892 and beginning of 1893. A comparison of these few results would seem to indicate that a correction of $-0''\cdot15$ has to be applied to observations by G to reduce to the standard of F, but the observations are too few to settle this point. A term G has therefore been introduced into the equations of condition for 1892, in order to represent the relative personal equation.

From all the equations of a group normals were formed in C, x, y, &c., and the value of C derived from its normal in terms of the other quantities was then substituted in the other normals. In this way each group gave six sub-normals in x, y, x', y', G and ρ. The mean of all the sub-normals in x was then taken for the final equation in x, and similarly for the other quantities.

The solution of these equations gave

$$x = +''\cdot240 \pm ''\cdot023 \qquad y = +''\cdot080 \pm ''\cdot024$$
$$x' = -\cdot135 \pm \cdot025 \qquad y' = -\cdot227 \pm \cdot025$$
$$G = +\cdot004 \pm \cdot017 \qquad \rho = +\cdot131 \pm \cdot020$$

The coefficient of the annual term comes out considerably larger than found by Chandler, but this series is hardly long enough to distinctly separate the 427 and 365 days periods.

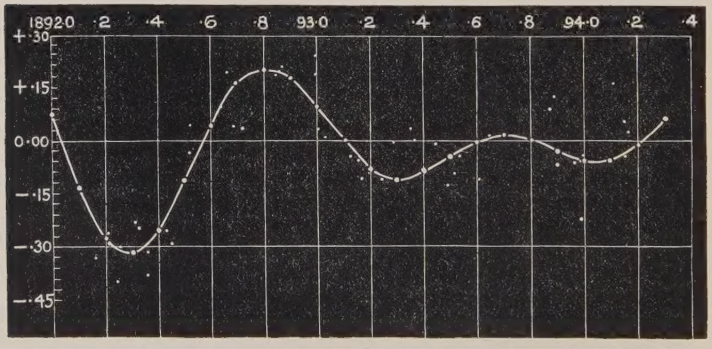
The probable error of a single equation (mean of three pairs on four nights) is $\pm$″·055 ; so that the probable observational error of a single latitude from one pair is $\pm$0″·168.

The following table gives the variations of latitude as computed from the above solution :—

TABLE I. (*values of* $\phi - \phi_0$)

1892·0	+″·076	1892·8	+″·207	1893·6	−″·001
·1	−·131	·9	+·184	·7	+·016
·2	−·277	1893·0	+·102	·8	+·001
·3	−·316	·1	+·004	·9	−·029
·4	−·253	·2	−·075	1894·0	−·054
·5	−·114	·3	−·098	·1	−·051
·6	+·043	·4	−·083	·2	−·010
·7	+·166	·5	−·042	·3	+·062

This curve is laid down on the accompanying diagram, and the dots show the separate results from the various groups.



The value of the aberration-constant from the above solution is larger than has been generally obtained, but the observations admit of this quantity being determined with almost entire freedom from error due to latitude-variation. There are twelve combinations of simultaneous evening and morning groups available for this purpose, but in a few cases there is a slight want of absolute coincidence in epoch. To allow for this a solution was made putting ρ=0, and the quantities in the fourth column of the following table have been applied to the absolute terms of the equations. These corrections are in every case very small and have no appreciable effect on the result.

The equations are :—

TABLE II.

Group.	Date.	No. of Nights.	$\delta d\phi$	Equation of Condition.	ρ
1	1892·19	10			
2	1892·22	8	−0″·28	$C_1-C_2+\ ·741\rho=-$″·004	+″·157
5	1892·35	10			
6	1892·35	8	·000	$C_5-C_6+\ ·955\rho=+·036$	+·188
2	1892·47	10			
3	1892·47	8	·000	$C_2-C_3+\ ·985\rho=+·348$	+·158
4	1892·68	9			
6	1892·67	9	−·010	$C_1-C_6-\ ·875\rho=-·276$	+·131
1	1892·86	6			
3	1892·86	8	·000	$C_1-C_3-\ ·849\rho=-·045$	+·154
4	1893·00	9			
5	1893·01	8	−·005	$C_4-C_5+\ ·767\rho=+·078$	+·110
1	1893·14	9			
2	1893·17	8	−·021	$C_1-C_2+\ ·761\rho=+·021$	+·185
5	1893·27	8			
6	1893·36	8	+·001	$C_5-C_6+\ ·930\rho=-·077$	+·071
2	1893·47	8			
3	1893·52	9	+·013	$C_2-C_3+1·035\rho=+·301$	+·106
1	1893·90	8			
3	1893·87	5	+·008	$C_1-C_3-\ ·793\rho=-·004$	+·113
4	1893·97	8			
5	1893·98	9	−·001	$C_4-C_5-\ ·710\rho=+·102$	+·152
1	1894·13	8			
2	1894 16	8	+·014	$C_1-C_2+\ ·748\rho=-·083$	+·050

Now the combinations of groups 1, 2 and 3 close, also those of groups 4, 5 and 6. Taking, then, these series independently, we have in the mean :—

TABLE III.

$C_1-C_2+0·750\rho=-$″·022 $\qquad$ $C_4-C_5+0·738\rho=+$″·090

$C_2-C_3+1·010\rho=+·324$ $\qquad$ $C_5-C_6+0·943\rho=-·021$

$C_3-C_1+0·821\rho=+·021$ $\qquad$ $C_6-C_4+0·875\rho=+·276$

therefore, by addition,

$2·581\rho=+·323$ $\qquad$ $2·556\rho=+·345$

and $\qquad \rho=+·125$ $\qquad$ $\rho=+·135$

In the second of these series there is only one combination of groups 6 and 4, so that slightly greater weight should be given to the result of the first.

Substituting $\rho = +0''.13$ in the equations of Table III. we find the values of $(C_1 - C_2)$ &c., and the substitution of these in Table II. leads to the separate values of ρ given in the last column of that table. The weighted mean of these is $+''.131$, and its probable error $\pm''.008$. These values of $(C_1 - C_2)$ &c. agree very closely with those found from the first general solution.

In the case of some of the stars the material available for the determination of proper motion is scanty, and it is quite possible that in a few cases the adopted motion may be $''.01$ or $''.02$ in error, but in the mean the result for ρ cannot be sensibly in error from this cause.

The constant of aberration, therefore, from this series of observations is $20''.57 \pm 0''.01$.

Royal Observatory, Cape of Good Hope:
1897 *October.*

Additional Note on Personal Equation. By Truman Henry Safford, Field Memorial Professor of Astronomy in Williams College.

In a former paper I have shown that the two-method personal equation is, on the average, about $0^s.13$ to $0^s.20$ for the ordinary time stars observed at Greenwich, and that it is persistently positive; or, in other words, that the average Greenwich observer anticipates by eye and ear the time of his own chronographic transits by an amount nearly equal to the "reaction-time" to a sense impression as determined by Wundt and other psychologists. To make rather more definite the astronomical results which I have obtained is the object of the present paper.

As the subject is one which involves two sciences, the research is one which it is difficult to put in logical order.

I will begin, then, by giving the average $e' - e$ for the years 1885 to 1894 inclusive, as derived from the Introductions to the Greenwich Observations. The averages are taken, giving each observer for the year equal weight.

Year.	Mean $e' - e$. s	No. of Observers.	Year.	Mean $e' - e$. s	No. of Observers.
1885	+0·158	5	1890	+0·180	5
1886	+0·145	6	1891	+0·122	5
1887	+0·170	5	1892	+0·165	10
1888	+0·212	5	1893	+0·123	15
1889	+0·162	5	1894	+0·162	10

The mean for the ten years is $0^s{\cdot}160$, or $0^s{\cdot}03$ more than that obtained by taking the mean of the two values given in my previous paper for Mr. Downing and Mr. Lewis separately.

For the "simple reaction" it is rather difficult to find a standard average value. Wundt, in the edition of 1880 of his *Grundzüge der physiologischen Psychologie*, gives the following as his own means from many experiments :—

	s
Reaction to sound	0·167
,, light	0·222
,, electric stimulation of the skin	0·201
,, sensation of touch	0·218

Hankel found

	s
Reaction to sound	0·1505
,, light	0·2246
,, electric stimulation	0·1546

The purely astronomical mean $0^s{\cdot}160$, obtained by combining the ten years' results obtainable from the Greenwich Observations, is sufficiently near the general average of these quantities to fully corroborate the conclusions of my previous paper, viz., that the phenomena of personal equation confirm the theories of Bessel and Wundt as to its psychical nature. In fact, we must be very careful to distinguish between the personal equations by the two methods, as they arise from quite different causes,* a fact sufficiently evident when we consider the amounts, $0^s{\cdot}5$ to $0^s{\cdot}6$, which the personal equation between the two methods can reach for the same observer.

The most puzzling form of personal equation seems to be that for faint stars observed by eye and ear ; and the study of that form by chronographic transits and by Bradley's method used side by side would seem to be a next step to be taken in the investigation of the entire subject.

I would call attention to some oversights in the MS. of my previous paper. In stating the polar equations of Romberg, Morine, Seraphimoff, Ditshénko and myself, the values $0^s{\cdot}026$, $0^s{\cdot}025$, $0^s{\cdot}042$ and $0^s{\cdot}006$ should be multiplied by tan δ or sec δ to give the actual time by which the observers named observe relatively too late or too early when the star is very near the pole.

Williamstown, Mass.:
1897 *November* 30.

* The variation of $e' - e$ from year to year is a marked feature for all, or nearly all, the observers, whether old or young, experienced or inexperienced. See, for example, p. 506 of my paper in the May number of the *Monthly Notices*.

Mean areas and heliographic latitudes of Sun-spots in the year 1895, deduced from photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lvii. p. 2, and are deduced from the measurements of solar photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn, India, and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1895, and Table II. gives the same particulars for the entire year 1895 and the six preceding years for the sake of comparison. The areas are given in two forms. First, projected areas—that is to say, as seen and measured on the photographs—these being expressed in millionths of the Sun's apparent disc ; and next, areas as corrected for fore-shortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1895 the mean daily area of whole spots, and the mean heliographic latitude of the spotted area for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots ; and Table IV. gives the same information for the year as a whole, similar results from 1889 to 1894 being added, as in the case of Table II. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888, on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*.

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TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected Whole Spots. Umbræ.	Faculæ.	Corrected for Foreshortening. Whole Spots. Umbræ.	Faculæ.
	d					
552	1895 Jan. 2·64	26	214 1208	1931	149 853	2137
553	Jan. 29·97	28	240 1296	1936	175 991	2136
554	Feb. 26·32	27	257 1552	1858	179 1114	2104
555	Mar. 25·63	27	183 1029	2085	138 812	2460
556	Apr. 21·91	27	211 1133	2224	158 853	2493
557	May 19·14	27	306 1764	2472	217 1280	2690
558	June 15·34	28	222 1294	2145	158 963	2380
559	July 12·54	27	217 1228	2010	143 833	2292
560	Aug. 8·76	27	267 1508	2177	195 1135	2420
561	Sept. 5·00	27	210 1206	2177	149 868	2399
562	Oct. 2·27	28	290 1641	2071	200 1175	2230
563	Oct. 29·56	27	231 1178	1793	163 851	1902
564	Nov. 25·86	27	194 1026	1847	142 787	1948

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Umbræ.	Projected Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1889	360	17·9	103	107	13·1	78·0	131
1890	361	21·3	133	273	15·5	99·4	304
1891	363	120	745	1322	86·2	569	1412
1892	362	255	1596	3230	186	1214	3270
1893	362	327	1983	2287	234	1464	2404
1894	364	317	1728	1666	231	1282	1877
1895	364	237	1330	2059	169	974	2278

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
	d.			°		°	°	°
552	1895 Jan. 2·64	26	392	12·51	461	14·19	− 1·91	13·42
553	Jan. 29·97	28	506	13·69	485	11·62	+ 1·30	12·68
554	Feb. 26·32	27	605	13·07	509	9·02	+ 2·97	11·22
555	Mar. 25·63	27	551	13·66	261	11·77	+ 5·48	13·05
556	Apr. 21·91	27	604	19·13	249	15·42	+ 9·05	18·05
557	May 19·14	27	686	16·41	594	16·68	+ 1·06	16·53
558	June 15·34	28	615	15·25	348	12·11	+ 5·37	14·12
559	July 12·54	27	302	12·80	531	9·00	− 1·11	10·38
560	Aug. 8·76	27	976	13·87	159	10·43	+ 10·46	13·38
561	Sept. 5·00	27	521	13·73	347	14·63	+ 2·39	14·09
562	Oct. 2·27	28	782	14·38	392	15·07	+ 4·54	14·61
563	Oct. 29·56	27	561	11·56	290	8·04	+ 4·88	10·36
564	Nov. 25·86	27	340	13·05	447	13·22	− 1·86	13·15

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1889	360	5·0	+ 7·26	73·0	− 11·90	− 10·68	11·61
1890	361	53·1	+ 22·20	46·3	− 21·75	+ 1·73	21·99
1891	363	401	+ 20·49	169	− 19·91	+ 8·52	20·31
1892	362	607	+ 15·09	607	− 21·69	− 3·29	18·39
1893	360	517	+ 14·91	941	− 14·26	− 3·93	14·49
1894	364	543	+ 12·31	739	− 15·56	− 3·75	14·18
1895	364	565	+ 14·26	409	− 12·54	+ 3·01	13·54

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

The Sun-spot record for 1895, as brought out by the above tables, shows some interesting points of comparison with the record for 1894.

(1) The decrease in the mean daily area of whole spots commenced in 1894 has been very distinctly continued in 1895; and this area has now fallen below those of 1882, 1883 or 1884, the years of maximum of the preceding cycle.

(2) The umbræ, which gave practically the same area for 1894 as for 1893, show for 1895 a diminution of 27 per cent.

(3) But the faculæ, which had declined very rapidly from their maximum in 1892, a year earlier than that of the spots, showed a distinct revival in 1895, their mean daily area for that year nearly rising to the same numbers as for 1893.

(4) Taking the two hemispheres separately, the decrease in the area of whole spots has been limited, as in 1894, to the southern hemisphere; the slight recovery of the northern having been further continued.

(5) In consequence of this decline of the southern hemisphere, the predominance has passed over to the northern.

(6) Little change has taken place in the mean distribution of all spots in heliographic latitude, but the general trend of movement is still towards the equator. But the mean distance from the equator is still greater than in 1883, the year of maximum of the preceding cycle.

(7) When the two hemispheres are considered separately, it is seen that this equatorial movement is wholly confined to the southern hemisphere; in the northern there has been a distinct increase in the mean latitude of the spotted area. Precisely the reverse conditions prevailed in 1894.

(8) No day in 1895 was entirely free from spots, although on November 10 but a single very small spot was seen.

Proper Motions of the three close Polar Stars Groombridge 1119, *Groombridge* 2283, *and Groombridge* 3548.

(*Communicated by the Astronomer Royal.*)

These three stars, as well as Bradley 1672 and Bradley 3147, were added to the Greenwich Clock Star list in 1896, for use in the determination of azimuth error. The stars have been very frequently observed at Greenwich during the years 1887–96, and the right ascensions and north polar distances deduced for the forthcoming new Ten Year Catalogue (1890) are taken as the standard places with which those of each of the other catalogues are compared. The proper motions of the two Bradley stars are given by Professor Auwers, and in order that the three Groombridge stars may be used for azimuth determination, it is necessary that their proper motions should also be well determined.

The right ascensions and north polar distances have been brought up to 1890 with Struve-Peters precessions, from Groombridge, Pond, Radcliffe, and as many later catalogues as were easily accessible. The precessions were computed by the trigonometrical method as given in Chauvenet's Astronomy, p. 615. The convenient arrangement of the computations given in the introduction of Carrington's Catalogue of Circumpolar Stars (1855) was adopted, and use made of the tables there given for facilitating the computation.

A correction to the right ascensions of $\frac{1}{4}\,\theta^2 \sin 2\alpha$, where θ denotes the angle between the mean equator of the date of each catalogue and the mean equator of 1890·0, was applied to the right ascensions, as this term is omitted in Carrington's formula. No systematic corrections have been applied to any of the catalogues used.

The right ascension and the north polar distance for 1890, as determined from each of the catalogues employed, was subtracted from the corresponding right ascension and north polar distance of the Greenwich ten year 1890. The proper motions deduced from each catalogue were combined with weights proportional to the product of the number of observations and the difference of epoch. The names of the catalogues, the mean dates of the observations, the number of observations, and the right ascension and north polar distance for 1890, (i) before applying proper motion, (ii) after applying proper motion, are given below.

The Greenwich right ascensions, north polar distances and proper motions for 1890·0, are

Name of Star.	R.A. 1890·0. h m s	P.M. s	N.P.D 1890·0. ° ′ ″	P.M. ″
Groomb. 1119	7 46 51·53	−0·1198	1 2 26·24	−0·020
Groomb. 2283	15 12 49·87	−0·0072	2 20 41·51	−0·031
Groomb. 3548	21 21 28·45	+0·0319	3 25 9·33	−0·018

Groombridge 1119.

Catalogue.		Mean Date of Observations.	No. of Observations.	R.A. 1890 Uncorrected for Proper Motion. h m 7 46 s	Secs. of R.A. 1890 Corrected for Proper Motion. s	Mean date of Observations.	No. of Observations.	N.P.D. 1890 Uncorrected for Proper Motion. ° ′ 1 2 ″	Secs. of N.P.D. 1890 Corrected for Proper Motion. ″
Groombridge	1810	1807·1	6	67·75	57·80	1807·1	9	28·65	26·99
Pond	1832	1832·1	4	63·61	56 66	1832·2	10	27·82	26·66
Radcliffe	1845	1850·6	70	56·59	51·86	1849·5	43	26·92	26·11
Paris	1845	1850·7	10	56·16	51·44	1847·1	2	27·0	26·14
Carrington	1855	1855·0	27	55·05	50·85	1855·0	27	26·77	26·07
Greenwich	1860	1857·1	5	52·73	48·78	1857·1	5	26·41	25·75
Paris	1860	1860·2	14	50·99	47·41	1856·9	41	26·6	25·94
Brussels	1865					1862·4	4	26·84	26·29
Harvard	1865	1863·7	33	53·89	50·73				
Greenwich	1872	1871·8	8	52·92	50·74	1873·3	9	26·46	26·13
Paris	1875	1877·3	33	51·43	49·91	1876·6	17	26·58	26·31
Greenwich	1880	1883·4	6	50·18	49·39	1882·6	13	26·35	26·20
Williams College	1885	1884·4	25	52·59	51·92				
Greenwich	1890	1893·0	419	51·17	51·53	1893·1	374	26·18	26·24

Proper Motion $\begin{cases} \text{R.A.} = -0^{s}\cdot1198 \\ \text{N.P.D.} = -0''\cdot020 \end{cases}$

Groombridge 2283.

Catalogue.		Mean Date of Observations.	No. of Observations.	R.A. 1890 Uncorrected for Proper Motion. h 15 m 12 s	Secs. of R.A. 1890 Corrected for Proper Motion. s	Mean date of Observations.	No. of Observations.	N.P.D. 1890 Uncorrected for Proper Motion. ° 2 ′ 20 ″	Secs. of N.P.D. 1890 Corrected for Proper Motion. ″
Groombridge	1810	1808·1	8	45·85	46·44	1808·1	8	44·59	42·05
Radcliffe	1845	1850·5	47	51·02	51·30	1848·9	11	42 20	40·93
Carrington	1855	1855·0	12	49·54	49·79	1855·0	12	42·20	41·11
Greenwich	1860	1860·3	2	49·98	50·19	1860·0	6	42·11	41·18
Greenwich	1864	1863·5	9	49·13	49·32	1864·0	12	42·58	41·77
Harvard	1865	1864·2	30	49·66	49·85				
Brussels	1865	1867·4	2	48·58	48·74	1864·5	2	41·77	40·98
Greenwich	1872					1868·6	5	43·11	42·45
Harvard	1875	1873·0	8	49·55	49·67	1873·0	6	42·75	42·22
Greenwich	1880	1886·8	3	49·61	49·63	1886 2	5	41·67	41·55
Williams College	1885	1885·3	22	49·48	49·51				
Greenwich	1890	1893·0	232	49·90	49·88	1893·1	205	41·41	41·51

Proper Motion $\begin{cases} \text{R.A.} = +0^{s}\cdot0072 \\ \text{N.P.D.} = -0''\cdot031 \end{cases}$.

Groombridge 3548.

Catalogue.		Mean Date of Observations.	No. of Observations.	R.A. 1890 Uncorrected for Proper Motion.	Secs. of R.A. 1890 Corrected for Proper Motion.	Mean date of Observations.	No. of Observations.	N.P.D. 1890 Uncorrected for Proper Motion.	Secs. of N.P.D. 1890 Corrected for Proper Motion.
				h m s	s			° ′ ″	″
				21 21				3 25	
Groombridge	1810	1807·8	7	25·41	28·01	1807·8	7	11·50	10·02
Radcliffe	1845	1850·9	70	27·20	28·45	1849·4	24	10·35	9·62
Carrington	1855	1855·0	19	27·71	28·83	1855·0	19	9·25	8·62
Brussels	1865	1864·5	6	27·58	28·39	1866·2	2	9·86	9·43
Greenwich	1867 & 1868	1867·7	2	28·14	28·85	1868·0	2	9·28	8·88
Harvard	1875	1873·5	10	27·77	27·98	1873·5	6	8·04	7·74
Greenwich	1880	1886·8	3	26·91	27·01	1886·5	7	9·49	9·43
Williams College	1885	1884·7	20	28·29	28·46				
Greenwich	1890	1894·2	155	28·54	28·44	1893·4	79	9·28	9·34

Proper Motion $\begin{cases} \text{R.A.} = +0^{s}\cdot0319 \\ \text{N.P.D.} = -0''\cdot018 \end{cases}$

The Binary Star h 5014. By R. T. A. Innes.

This star is identical with Piazzi 17^h 341, R.A. 17^h 59^m 36^s Dec. $-43°$ 25′·8 (1900) mag. 5·2 from Bailey's Southern Photometry. Its motion was early recognised, but the measures of Jacob, which we can now see are mutually inconsistent, made it difficult to reconcile all the observations made into even a passable orbit. It will be understood that from Jacob's station the star can only be seen at a comparatively small altitude, and from the closeness of the components and the inferiority of the telescope used it must have been a very difficult pair to deal with. The components are very nearly equal in magnitude ; on one occasion Jacob found half a magnitude of difference between them, and on several occasions I have thought the now preceding star slightly the fainter, but it is really doubtful if it is so. I have, however, added 180° to all angles since the time of Jacob, and get thus :—

	Angle.	Distance.		Nights.
	°	″		
1836·7	69·1	0·67	*h*	2–1
1856·7	312·3	0·5 ±	Jacob	1
1857·7	317·2	0·6 ±	„	1
1878·7	268·0	1·38	Melbourne	1
1880·5	259·3	0·81	Russell	1
1886·6	254·8	1·27	Pollock	1
1887·8	253·0	1·38	„	3–2
1893·6	248·1	1·02	Sellors	3
1895·6	247·3	1·47	„	3
1896·6	245·6	1·49	„	3

Some of the measures of distance seem to suffer from large errors of observation. The motion is retrograde. It will be seen that apparent periastron probably took place about 1840–1855, and that in fifty-seven years from the date of discovery half of the angular orbit was described. It looks, however, as if the period was much in excess of twice the number of years already elapsed.

The Melbourne measure, which was probably made by Mr. Ellery, was kindly communicated by Mr. Baracchi, the present Government Astronomer there.

The series of measures made under Mr. Russell's direction is very valuable. This pair has a common proper motion of 0″·14 per annum towards 208°·6.

One of *h*'s measures is set against *h* 5013, and his identification of the star as *Brisbane* 6308 is also erroneous. The star *Brisbane* 6308 follows and is included with the pair in a low power field.

Royal Observatory, Cape of Good Hope:
1897 *November* 17.

Occultation of Ceres by the Moon on 1897 *November* 13. *Observed at the Hamburg Observatory.* By Prof. George Rümker, M.A., Director of the Observatory.

(*Communicated by the Secretaries.*)

In consequence of the communication in *Monthly Notices*, vol. lvii. No. 9, it has been possible for Dr. R. Schorr to observe the occultation with the $9\frac{1}{2}$-inch equatorial of this observatory. The planet was clearly visible before the immersion near the Moon's limb, but, unfortunately, a few seconds before occultation thin clouds covered the Moon, and the planet disappeared. The time of immersion could therefore only be estimated as $10^h\ 45^m\ 15^s$ Greenwich Mean Time, with an uncertainty of ± 5 seconds. The reappearance of the planet was well observed in a cloudless sky at $11^h\ 43^m\ 4^s{\cdot}4$ Greenwich Mean Time.

The reappearance was not instantaneous, but the light showed a distinct increase for $0^s{\cdot}1$ or $0^s{\cdot}2$. With a comet-seeker of 5 inches clear aperture the phenomena could not be observed.

Occultation of Ceres by the Moon, 1897 *November* 13, *observed at the Radcliffe Observatory, Oxford.*

(*Communicated by the Radcliffe Observer.*)

The occultation-reappearance of *Ceres* (on the Moon's dark limb) was observed here by Mr. Wickham and myself as follows :

	Observer.	Instrument.	Power.	Time Noted. h m s	Oxford Sidereal Time. h m s	Greenwich Mean Time. h m s
1897 Nov. 13	A. R.	Barclay Equat.	45	2 58 25·5	2 58 11·98	11 29 54·6
	W.	Heliometer	80	2 59 0·2	2 58 12·03	11 29 54·7

Observers' Remarks : Instantaneous (A. R.). Very good (W.).

Radcliffe Observatory : 1897 *December* 9.

Observations of Meteors on 1897 *November* 13–15, *made at the Radcliffe Observatory, Oxford.*

(*Communicated by the Radcliffe Observer.*)

On November 13 the watch for meteors was continuous from $11^h\ 15^m$ till $17^h\ 45^m$ G.M.T., when clouds quickly and permanently covered the sky.

The night on the whole was very fine, with a heavy dew. The atmosphere was very clear throughout, and the sky cloudless, except for a few short intervals.

The observers' position on the east wing of the Observatory tower commanded an uninterrupted view of the eastern half of the sky. The Moon was, however, very bright, but fortunately the observers were able to work in the shadow of the tower during the greater part of the watch.

Mr. Wickham reports :—A star chart of the region had been previously plotted, and each observer had provided himself with duplicate copies. Mr. Robinson began a watch of the eastern sky at $11^h\ 15^m$, but had only noted one sporadic meteor before he was joined about midnight by Dr. Rambaut and myself, who had been detained by the observation of the occultation of Ceres by the Moon. The number of meteors seen amounted to about forty ; some were quite out of the track of the *Leonids*, and a few were too faint and rapid to leave a definite impression of their paths upon the mind. The presence of the Moon proved a serious obstacle to seeing the fainter meteors, stars less than $4\frac{1}{2}$ mag. being invisible, whilst, in the actual neighbourhood of the Moon, bright stars were recognised only with difficulty. The cluster *Præsepe* could be occasionally seen by glimpses when on the meridian.

Generally, the *Leonids* charted were fainter than the 3rd mag., with a rapidly evanescent track and a duration of about two-tenths of a second only. When bright enough for comparison with star magnitudes, notes of the estimation were made.

The rather numerous sporadic meteors were generally much brighter than the *Leonids*, and had longer and more lasting trails.

General notes by Mr. Robinson :—Stars down to mag. 4 were seen without difficulty, and others slightly fainter could be traced at times. *Præsepe* was visible at intervals, but only as a very feeble haze. The method of observation I adopted was as follows :—Immediately on the appearance of a meteor, a 13-inch pencil, held firmly at arm's length, was placed along the meteor's path, the direction, length of track, and then the approximate time estimated, and the track pencilled on a prepared chart. The chart used contained stars, down to the 4th magnitude inclusive, in *Leo* and surrounding region. A few of the meteors were not sufficiently well observed for an accurate delineation of their paths, but those recorded in the chart are considered fairly accurate in direction, although the swiftness of the phenomena rendered observations very difficult.

The meteors were generally between the 2nd and 3rd magnitude, the brightest, which excelled the 1st magnitude, occurring at $16^h\ 32^m$ G.M.T. Some as faint as mag. 4 were recorded, but any beyond this limit would, in most cases, be obliterated by the moonlight. Several meteors were obviously from radiants other than *Leo*, but their positions were as a rule only approximately secured.

Radcliffe Observatory, Oxford. Observation of Meteors, 1897 November 13.

Ref. No.	Approx. G.M.T. h m	Observers.	Observers' Notes.
1	11 42	R.	Through Castor and Pollux towards equator.
2	12 27	A. R: W.	(W.) A short flash, duration 0ˢ·3.
3	12 44	A. R: W: R.	(A. R.) Time not observed; faint trail. (W.) Not charted; faint streak; 1 sec. (R.) Beads; good observation.
4	12 57	W.	Not charted; in Ursa Maj.
5	13 5	A. R: R.	(A. R.) Swift.
6	13 16	A. R.	Sporadic; through Ursa Maj.
	13 24		(Clouds passing in south.)
7	13 27	A. R: W: R.	(A. R.) Very swift; faint. (R.) Charted on the morning of the 15th from sketch and notes made at the time of observation.
8	13 32	R.	Sporadic in Ursa Maj.
	13 34		(Clouds over Leo.)
9	13 47	A. R: W: R.	(A. R.) Faint. (R.) Sporadic.
10	14 5	R.	Swift.
11	14 7	A. R: W: R.	Sporadic in Ursa Maj. (R.) Very bright.
12	14 21	W.	Sporadic in Ursa Maj.
13	14 22	W.	
14	14 23	W.	Brighter than α Hydræ.
15	14 26	W: R.	(W.) 2nd mag. (R.) 1st mag. Track approximate only.
16	14 37	R.	Across Leo; 4th mag.; swift.
17	14 42	R.	(R.) 4th mag. From Sickle to north of β Leonis.
18	14 59	R.	Sporadic.
19	15 1	W.	
20	15 4	R.	From Sickle to Castor; 4th mag.
21	15 29	W: R.	(R.) Only partially seen.
	15 40 to 15 45		(Cloudy; watched in breaks.)
22	15 47	R.	Sporadic; Capella to Sickle; 2nd mag.
23	15 52	W: R.	(W.) Brighter than 2nd mag. (R.) 2nd mag. good.
24	15 57	W: R.	Sporadic from south of Leo. (R.) 2nd mag.
25	15 59	R.	Sporadic; 3rd mag.
26	16 14	W: R.	(W.) Very swift and faint. (R.) Only approximate; very swift; $2\frac{1}{2}$ mag.
27	16 32	W: R.	(W.) Similar light to magnesium; brighter than 1st mag. (R.) Brighter than 1st mag.
28	16 47	W: R.	(W.) Streak. (R.) Train; several secs. duration.

Ref. No.	Approx. G.M.T. h	m	Observers.	Observers' Notes.
29	17	3	W: R.	(R.) Charted on the morning of the 15th from sketch and notes made at the time of observation.
30	17	3½	W: R.	(R.) Shorter track and slightly greater altitude than last.
31	17	11	W.	ζ Cancri to below Procyon.
32	17	17	W: R.	(W.) Very swift. (R.) Seen, but not charted; very swift; from about 10h 30m R.A. Decl. +30° towards Arcturus.
33	17	32	R.	Approximate; very swift; 3rd mag.
34	17	39	W: R.	(R.) 2nd mag.
35	17	39	W.	Sporadic.
	17	45		(Very cloudy.)

Observers: A. R., A. A. Rambaut. W., W. Wickham. R., W. H. Robinson.

Meteor Tracks, 1897 Nov. 13.
Observers: A. A. Rambaut ——— W. Wickham ———
W. H. Robinson .....

The night of November 14 was overcast, with frequent rain, but on the 15th, although the sky was generally cloudy, Mr. Robinson reports that frequent breaks occurred in the neighbourhood of *Leo* between 12h 45m and 15h 30m, during which interval

only one meteor was seen at $13^h\ 9^m$ G.M.T., its track proceeding from $5^h\ 50^m$ R.A. $+40°$ Decl. to $3^h\ 40^m$ R.A. $+25°$ Decl. During the three hours' watch opportunities enough were afforded to have noted anything like a rich shower had it occurred.

On the effect of Chromatic Dispersion of the Atmosphere on the Parallaxes of α Centauri and β Orionis, and on a method of determining its effect on the Value of the Solar Parallax derived from Heliometer Observations of Minor Planets. By David Gill, C.B., LL.D., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope.

In connection with the discussion of my Heliometer Observations of *Mars* in 1877 (*Mem. R.A.S.* vol. xlvi. pp. 121 and 161), I have called attention to the effect which a difference between the mean refrangibility of the light of *Mars* and that of the comparison stars might have upon the resulting value of the solar parallax.

In the determination of diurnal parallax it is unfortunately impossible to distinguish with sufficient accuracy between the refraction effect in question and a small residual error in the parallax, because, within such range of zenith distance as accurate observations can be made, the law of increase of refraction does not sufficiently differ from that of parallax.

In observations for stellar parallax it may, with proper precautions, be possible to determine the effect of different refrangibility in the mean light of two stars, because measures of distance can be made at widely different hour angles E and W of the meridian, in which such difference of refraction has opposite effects on the measured distance, whilst the parallax factor remains the same.

Professor Rambaut (*Monthly Notices R.A.S.* 1895 January, vol. lv. p. 123) discusses several series of observations of distance measures of the same pairs of stars made at widely different hour angles, and in some instances obtains a certain amount of evidence that the difference of the mean refrangibility of the light of the two stars is sufficiently great to produce in filar micrometer measures a very sensible error in the derived mean distance, if no corresponding correction is employed in computing the differential refraction.

The only difficulty in computing such corrections is to ascertain what precise wave-length (in the very short spectrum produced by atmospheric dispersion) is bisected by the observer in his observations of any particular star. Indeed, in the case of heliometer observations of the distance between two stars it is not improbable that in estimating the coincidence of the two very short spectra produced by chromatic dispersion of the atmosphere

the observer may bring the two *similarly coloured*, and not the two *brightest parts* of the star spectra into coincidence. If this be the case no correction depending upon the difference of the refrangibility of the "mean light" of the two stars would be required in heliometer observations.

The length of the visible spectrum produced by atmospheric dispersion for fairly bright stars is estimated by Airy to be $\frac{1}{60}$th part of the total refraction; or

	Length of Spectrum.
For $\zeta = 46^\circ$	1″·0
57	1·5
64·3	2·0

and very refined observations cannot be made at greater zenith distances than 64° or 65°. It is obvious that, in such short spectra, the observer cannot say with certainty whether he brings into coincidence the brightest parts of the images or the parts which are similarly coloured, and two different observers may even have different habits of observation in this respect or may involuntarily make some compromise between the two methods. The only available test seems to be the actual measurements of angles under widely different conditions of refraction.

The whole question is one of very great importance, especially in connection with recent observations of minor planets for the determination of the solar parallax; and, as the observations of the planets themselves are not available for the purpose, for reasons already stated, any reliable results that can be derived from observations of stars are of special interest and importance. I also entirely concur in Professor Rambaut's conclusion that it is desirable to discuss the existence or otherwise of such a source of possible error in all refined observations for stellar parallax. But I cannot concur in the conclusions which he has drawn from his rediscussion of my own observations for the parallax of *α Centauri*.

This rediscussion is based on the fact that I found it necessary to introduce four unknown quantities in the equations of condition to express the effects of certain subjective systematic errors, and on Professor Rambaut's belief that by the introduction of a single term (expressing the effect of a difference between the mean refrangibility of the light of *α Centauri* and that of the comparison stars) a more simple and probably more correct solution is arrived at.

In the heliometer observations of *α Centauri* in question no reversing prism was employed to eliminate the possible subjective effect of the direction of measurement with respect to the vertical.* In my own experience, and I believe in that of

* The reversing prism was found to absorb an inconveniently large amount of light in the observation of the comparatively faint comparison stars as viewed through the semi-lens of a micrometer of only 4 inches aperture.

others also, it is found necessary, when working without the reversing prism, to turn the observer's head, so that the line joining the two stars under measurement is either parallel to the line joining the observer's eyes or at right angles to that line; without this precaution I am unable to estimate accurately the symmetrical passing and repassing of the star discs through each other when the position-angle handle is moved in opposite directions in the process of "crossing through." During the observations of α_2 *Centauri* I had an instinctive feeling that the pointings might be influenced to some extent by the neighbourhood of the comparison star α *Centauri*, and that this effect would be different according as the comparison appeared to the right or left, or above or below, the line joining the observer's eyes. Therefore, in the first solution, four terms, $x_{,}$, $x_{,,}$, $x_{,,,}$, $x_{,,,,}$, were introduced to represent the four subjective errors which might be expected under the four different conditions of measurement.

But in order to determine whether these assumed subjective errors are really constants, or whether they change gradually with the hour angle, a second solution was made, in which the subjective errors were represented as a periodic function of $(p-q)$, and, instead of $x_{,}$, or $x_{,,}$, or $x_{,,,}$, or $x_{,,,,}$, there was substituted in every equation

$$x + \cos(p-q)\,\alpha + \sin(p-q)\,\beta + \cos 2(p-q)\,\gamma + \sin 2(p-q)\,\delta,$$

where p is the position angle and q the parallactic angle.

These two solutions gave the following results :—

Solution I.	*Solution II.*
$x_{,}, x_{,,}, x_{,,,}, x_{,,,,}$ assumed constants.	Systematic corrections assumed a function of $(p-q)$.
$\pi = 0''.747$ p.e. $\pm 0''.0133$	$\pi = 0''.756$ p.e. $\pm 0''.0140$
Prob. error of 1 observation $\pm 0''.1181$	Prob. error of 1 observation $\pm 0''.1340$

Thus the weight of an observation by Solution II. is reduced in the proportion of $(0''.1181)^2 : (0''.1340)^2$, or of 1 : 1·3.

The hypothesis that the systematic errors are really subjective constants (corresponding to the four directions of the line joining the observer's eyes with respect to the direction of measurement), and not a function of the hour angle, is therefore the more probable.

Professor Rambaut in his paper does not refer to this Solution II., nor to the conclusions derived from it, but he makes another solution (which we shall call Solution III.), in which the possibility of the existence of systematic errors is ignored, and a single term x, whose coefficient is $\tan\zeta \cdot \cos(p-q)$, is introduced in place of $x_{,}$, or $x_{,,}$, or $x_{,,,}$, or $x_{,,,,}$.

The origin of this term is as follows :—

The astronomical refraction of a star of normal colour may be represented by

$$R_0 = \beta_0 \tan \zeta,$$

where ζ is the zenith distance and β_0 is a function of ζ.

For another star whose mean light is of different refrangibility we should have

$$R' = (\beta_0 + \Delta\beta) \tan \zeta.$$

In correcting any instrumental measure of the angular distance between two stars for the effects of refraction by the usual formulæ for differential refraction we take into account the term β_0 for both stars, but we omit entirely the effect of refraction depending on $\Delta\beta$; and, as the abnormally coloured star would, under identical conditions with the normally coloured star, be displaced vertically by

$$\Delta\beta \,.\, \tan \zeta,$$

the refraction correction in distance computed by the ordinary differential formulæ must be supplemented by the farther correction

$$\Delta\beta \,.\, \tan \zeta \cos (p - q).$$

Employing precisely the same observations as those used in my Solution I., Professor Rambaut finds

Solution III.

$$\pi = 0''{\cdot}780 \pm 0''{\cdot}018$$

$$x = \Delta\beta = \overset{R}{+0{\cdot}0074} \pm \overset{R}{0{\cdot}0023} = \overset{''}{+0{\cdot}095} \pm \overset{''}{0{\cdot}029},$$

and for the probable error of one observation $\pm 0''{\cdot}1686$.

If this Solution III. be a true one, it involves a most serious question in practical astronomy. Of the two comparison stars employed that named a was practically identical in colour and magnitude with α_2 *Centauri*, as viewed through the wire gauze screen which was employed in the observations; the star β is of a slightly reddish colour, and also slightly fainter than the reduced image of α_2 *Centauri* (*Mem. R.A.S.* vol. xlviii. p. 16). The value of $\Delta\beta$ found from Solution III. implies that the refraction for the principal star relative to the mean of that of the two comparison stars requires a correction of

$$+0''{\cdot}095 \,.\, \tan \zeta.$$

But as one of the stars is of the same colour as α_2 *Centauri*, and thence its mean refrangibility probably the same, we should have to attribute the whole of this abnormal refraction to the abnormal colour of the star β, and hence the abnormal refraction of star β would amount to

$$+0''{\cdot}19 \tan \zeta.$$

If this be true the apparent parallactic displacement of a minor planet of the same colour as the star β would be affected

by an error, due to abnormal refraction, of $0''{\cdot}19$ at zenith distance 45°, or of $0''{\cdot}38$ at zenith distance 64°.

It is necessary to examine Professor Rambaut's solution very thoroughly before so alarming a conclusion can be accepted.

In the first place it is a suspicious fact that the weight of an observation by Solution III. is reduced to less than one-half of its weight, according to Solution I., viz. in the proportion of $(0{\cdot}1181)^2 : (0{\cdot}1685)^2$.

Professor Rambaut attributes the very inferior representation of the observations which he obtains to the smaller number of unknown quantities which he employs, viz. four, instead of seven employed by me. This might be a valid explanation if the number of equations of condition was comparatively small ; but as they are ninety-four in number, and the "sum of the squares of the residuals" has been divided, *not* by 94, but by $(94-4)$ in the case of Professor Rambaut's results, and by $(94-7)$ in the case of my own, the resulting values of the "square of the mean error" are strictly comparable.

The introduction of additional unknown quantities whose factors represent the laws of existing sources of objective or subjective error will certainly reduce the sum of the squares of the residuals ; but the exclusion of terms which represent real quantities, or the introduction of terms which have no real values, must tend to increase the resulting probable error.

(A very apt practical illustration of these facts will be found in connection with the discussion of the parallax of *β Orionis* contained in a subsequent part of the present paper.)

To render subsequent discussion clear, let us suppose that a series of observations for parallax has been made in six groups, as follows :—

Group.

a Observations at first parallax maximum, hour angle E.

b Observations at first parallax minimum, hour angle W.

c Observations at first parallax zero, hour angle E.

d Observations at first parallax zero, hour angle W.

e Observations at second parallax maximum, hour angle E.

f Observations at second parallax minimum, hour angle W.

Let us farther suppose that all the observations of groups *a*, *c*, and *e* have been made at precisely the same hour angle east of the meridian, and all the observations of groups *b*, *d*, and *f* at another constant hour angle west of the meridian ; we then have equations of the following types :—

Group.

$$a \quad x_E + (T - t_a)y + f_E \,.\, \Delta\beta + f'\pi = n_a$$

$$b \quad x_W + (T - t_b)y + f_W \,.\, \Delta\beta - f'\pi = n_b$$

$$c \quad x_E + (T - t_c)y + f_E \,.\, \Delta\beta \qquad = n_c$$

$$d \quad x_W + (T - t_d)y + f_W \,.\, \Delta\beta \qquad = n_d$$

$$e \quad x_E + (T - t_e)y + f_E \,.\, \Delta\beta + f'\pi = n_e$$

$$f \quad x_W + (T - t_f)y + f_W \,.\, \Delta\beta - f'\pi = n_f$$

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where x_E and x_W represent such systematic subjective error as may be due to the inclination to the vertical or the position of the observer's head, &c.

T is the mean epoch of all the observations ; t_a, t_b, &c., the epoch of the groups *a*, *b*, &c.

f_E and f_W the factors of $\Delta\beta$ (already defined); and, as the hour angles of all observations of the same class are identical, these factors are simply constants.

f' is the parallax factor at maximum.

n_a, n_b, &c., the absolute terms of the equations.

From these six equations, or from any number of similar equations, we can determine y, π, $(x_E+f_E\Delta\beta)$, and $(x_W+f_W\Delta\beta)$; but it is impossible to obtain the separate values of x_E, x_W, and $\Delta\beta$, because the relations of their coefficients to each other are the same in every equation in which they occur.

In order to obtain the separate value of $\Delta\beta$ it would be necessary to make observations both E. and W. of the meridian at very different hour angles, so as to vary the values f_E and f_W (which then are no longer constants, but functions of the hour angle); but by so doing we introduce the possibility of other subjective errors of the kind already described unless some method of observing (such as the use of the reversing prism) is employed to eliminate the effect of subjective errors depending on the inclination of the direction of measurement.

The most powerful method of determining stellar parallax with or without the reversing prism is therefore to adopt the plan of making all the observations at one or other of two definite hour angles, and to include in the series, not only observations at the maximum and minimum of parallax, but also a series of observations at both hour angles, the observations in which series are to be made nearly about the same epoch (*i.e.* when the parallax coefficient is nearly the same for the E. and W. observations).

We have, then, only one systematic error to discuss, viz. (x_E-x_W), and we obtain it in the simplest and most powerful manner possible, only we are unable to say whether the error so determined has its origin in subjective causes (such as some form of personality) or in objective causes (such as difference in the colour of the two stars under observation).

In a nearly continuous series of observations like that under discussion it would not be possible to make all the observations at two hour angles only, but we might make them at four different selected hour angles, in which case we should have to introduce into each equation $x_{/}$, or $x_{//}$, or $x_{///}$, or $x_{////}$ to represent the systematic error (subjective or objective) due to the particular hour angle at which the observation was made. Here, again, the solution would be absolutely free from systematic error as regards the determination of π, but would not give any information as to whether x, $x_{//}$, $x_{///}$, and $x_{////}$ were of subjective or objective origin.

In the observations of *α Centauri* I had assumed that the

equations resulting from the observations could be divided into four classes, viz.—

Equations involving	Sidereal Time. h m to h m	
$x_{,}$	8 7 to 10 29	direction of measurement parallel to observer's eyes
$x_{,,}$	11 15 to 13 9	direction of measurement at right angles to observer's eyes
$x_{,,,}$	15 50 to 19 38	direction of measurement parallel to observer's eyes, but reversed 180°
$x_{,,,,}$	19 50 to 21 41	direction of measurement at right angles to observer's eyes, but reversed 180°

and that the subjective errors for each of these classes is constant throughout the range of that class.

Now, the point at issue between Professor Rambaut and myself is whether the true systematic errors consist of four constants, due to the four different conditions of measurement, or whether they arise from the omission on my part of a single term, $\Delta\beta$, whose coefficient is $\cos(p-q)\tan\zeta$.

The residuals from Professor Rambaut's solution and my own, divided into four groups according to the direction of measurement with respect to the line joining the observer's eyes, as described above and arranged in each group in order of the sidereal time of observation, are given in the following tables :—

RESIDUALS.

From Equations in $x_{,}$.

Rotation Number.	Sid. Time. h m	Cos $(p-q)$ × tan ζ.	Residuals. Gill. R	Residuals. Rambaut. R	Weight.
22	8 7	+0·49	−0·028	−0·012	½
23	8 52	+0·64	− 19	− 5	½
47	8 52	+0·64	− 1	+ 10	
30	8 56	+0·65	+ 6	+ 17	
33	8 58	+0·65	− 11	− 1	
24	9 4	+0·66	− 22	− 9	
37	9 6	+0·67	+ 4	+ 13	
27	9 7	+0·67	− 15	− 3	
46	9 8	+0·67	+ 19	+ 19	
45	9 9	+0·67	+ 23	+ 33	
32	9 10	+0·68	+ 7	+ 17	
42	9 17	+0·68	− 5	+ 5	
25	9 24	+0·69	− 7	+ 6	
34	9 31	+0·70	− 12	− 3	
35	9 31	+0·70	+ 1	+ 11	
52	9 41	+0·71	− 6	+ 5	

Rotation Number.	Sid. Time. h m	Cos $(p-q)$ × tan ζ.	Residuals. Gill. R	Rambaut. R	Weight.
39	9 45	+0·71	+0·002	+0·011½	
26	9 49	+0·71	− 16	− 4	
56*a*	9 58	+0·72	+ 14	+ 27	
88	10 4	+0·72	− 6	+ 7	½
64	10 7	+0·72	+ 12	+ 26	½
91	10 9	+0·72	+ 7	+ 21	
50	10 12	+0·72	− 7	+ 3	
63	10 21	+0·72	− 1	+ 13	
104	10 24	+0·72	+ 15	+ 32	
98	10 25	+0·72	+ 13	+ 28	
61	10 29	+0·73	+ 13	+ 28	
		From Equations in x_{II}.			
60	10 19	+0·72	+0·014	+0·010	
95	11 3	+0·71	− 5	− 8	
38	11 23	+0·70	+ 9	0	
97	11 28	+0·70	− 52	− 55	½
103	11 44	+0·68	+ 13	+ 13	
90	11 46	+0·68	+ 3	− 1	
54	11 47	+0·69	+ 5	− 1	
40	11 53	+0·69	− 11	− 10	
101	11 56	+0·67	+ 14	+ 13	
59	11 56	+0·67	− 1	− 5	
55	11 55	+0·67	+ 14	+ 9	½
43	12 7	+0·66	+ 3	− 11	
31	12 47	+0·60	− 7	− 15	
36	13 9	+0·57	− 8	− 16	
		From Equations in x_{III}.			
2	15 50	+0·18	+0·005	−0·017	
1	16 30	+0·04	+ 16	− 4	
44	16 42	0·00	+ 13	− 11	
5	16 42	−0·04	0	− 19	
7	16 50	−0·04	+ 10	− 8	
76	17 19	−0·16	+ 4	− 8	½
105	17 24	−0·18	+ 9	− 6	
10	17 26	−0·19	+ 8	− 7	
74	17 29	−0·20	− 35	− 47	½
96	17 35	−0·23	− 8	− 25	½
93	17 45	−0·28	− 32	− 49	½

Rotation Number.	Sid. Time. h m	Cos $(p-q)$ × tan ζ.	Residuals. Gill. R	Rambaut. R	Weight.
62	17 45	−0·28	−0·015	−0·034	
92	17 51	−0·31	− 13	− 30	
73	17 53	−0·32	+ 4	− 9	½
102	18 5	−0·38	− 25	− 39	
4	18 6	−0·38	+ 4	− 13	
9	18 7	−0·39	− 2	− 16	
94	18 8	−0·39	+ 3	− 13	
81	18 9	−0·40	− 22	− 32	½
49	18 17	−0·44	+ 18	− 2	
3	18 25	−0·49	− 12	− 28	
79	18 29	−0·51	+ 2	+ 7	
100	18 29	−0·51	− 2	− 16	
51	18 37	−0·56	0	− 19	
13	18 37	−0·56	+ 19	+ 7	
78	18 38	−0·57	+ 2	− 7	½
53	18 40	−0·58	+ 2	− 17	
6	18 42	−0·59	− 4	− 19	
68	18 42	−0·59	− 6	− 19	
82	18 43	−0·60	+ 4	− 4	
11	18 49	−0·63	+ 9	− 3	
83	18 55	−0·67	− 14	− 21	
84	19 1	−0·71	+ 2	− 5	
14	19 7	−0·75	− 16	− 28	
70	19 10	−0·77	+ 16	+ 5	
71	19 16	−0·82	− 2	− 12	
12	19 20	−0·84	+ 9	− 1	
80	19 21	−0·85	− 3	− 10	
67	19 25	−0·88	+ 5	− 5	
99	19 35	−0·96	− 4	− 25	
58	19 38	−0·98	+ 13	0	

From Equations in x_{IIII}.

85	19 50	−1·08	−0·008	+0·018	½
17	20 4	−1·20	+ 14	+ 36	
69	20 10	−1·26	− 7	+ 17	
19	20 11	−1·27	− 15	+ 8	
87	20 21	−1·37	+ 39	+ 67	
57	20 22	−1·38	+ 20	+ 40	

Rotation Number.	Sid. Time. h m	Cos $(p-q)$ × tan ζ.	Residuals. Gill. R	Residuals. Rambaut. R	Weight.
56	20 27	−1·43	−0·002	+0·018	
86	20 39	−1·55	− ·23	+ 5	
72	20 41	−1·58	− ·11	+ 15	
20	20 45	−1·62	− ·38	− 15	
75	21 1	−1·82	+ ·25	+ 55	½
21	21 3	−1·83	+ ·3	+ 28	
18	21 41	−2·39	+ ·9	+ 40	

A single glance at these tables is sufficient to show that Professor Rambaut's solution gives no adequate representation of the observations.

Of his twenty-seven residuals from the equations in $x_{,}$ only seven are negative.

Of his fourteen residuals from the equations in $x_{,,}$ only four are positive.

Of his forty-one residuals from the equations in $x_{,,,}$ only three are positive.

Of the thirteen residuals from the equations in $x_{,,,,}$ only one is negative; and in every case all his exceptional residuals are very small.

The means of the residuals of the four groups are :—

From equations in	$x_{,}$ R	$x_{,,}$ R	$x_{,,,}$ R	$x_{,,,,}$ R
Gill	−0·0001	+0·0008	+0·0001	+0·0002
Rambaut	+0·0111	− ·0042	− ·0139	+ ·0246

In fact, so far from eliminating my systematic errors, Rambaut's mean residuals for the four different classes of observation are, on the whole, actually larger than the systematic corrections found by me, viz.—

$x_{,}$ R	$x_{,,}$ R	$x_{,,,}$ R	$x_{,,,,}$ R
+0·0178	−0·0003	−0·0214	+0·0093

and, most remarkable of all, his representation of Group $x_{,,,,}$, where the coefficients cos $(p-q)$ tan ζ are largest, and where, therefore, his correcting term should have the largest correcting influence, is the group which is worst represented, and has the largest mean residual. Solution I., on the other hand, represents the observations with remarkable accuracy, and the + and − residuals are, on the whole, very evenly distributed.

It has already been shown that if all the observations of the same group had been made at the same hour angle, and the four groups had been observed at four different hour angles, we should, by the introduction of our four unknown quantities, have a determination of π absolutely free from suspicion of systematic error; but there would be no means of distinguishing between subjective error and error depending on $\Delta\beta$.

But as in each of the four groups the observations extend over a considerable range of hour angle, it remains to determine whether $x_{,}$, $x_{,,}$, &c. vary with the hour angle in such a way as to point to a real effect depending upon $\Delta\beta$.

No conclusions can be drawn from mere inspection of the residuals. Rambaut (*loc. cit.*) has given the value of

$$\cos(p-q)\tan\zeta,$$

which is the coefficient of $\Delta\beta$, and accordingly with these coefficients and the original equations the terms in the normals and the normal equation in $\Delta\beta$ have been computed. The complete normals are :—

$+24{\cdot}50x_{,}$				$+\ 8{\cdot}04y$	$+\ 6{\cdot}86y'$	$+15{\cdot}99z$	$+16{\cdot}84\Delta\beta$	$=+2{\cdot}16^{R}$
	$+13{\cdot}00x_{,,}$			$+\ 7{\cdot}99$	$+\ 8{\cdot}35$	$+\ 9{\cdot}32$	$+\ 8{\cdot}72$	$=+9{\cdot}99$
		$+37{\cdot}00x_{,,,}$		$+12{\cdot}61$	$+18{\cdot}82$	$-\ 8{\cdot}69$	$-17{\cdot}64$	$=-1{\cdot}76$
			$+12{\cdot}00x_{,,,,}$	$+\ 2{\cdot}71$	$+\ 2{\cdot}53$	$-\ 6{\cdot}64$	$-18{\cdot}33$	$=-0{\cdot}62$
$+\ 8{\cdot}04$	$+\ 7{\cdot}99$	$+12{\cdot}61$	$+\ 2{\cdot}71$	$+36{\cdot}56$	$+37{\cdot}32$	$+14{\cdot}95$	$+\ 0{\cdot}19$	$=+1{\cdot}42$
$+\ 6{\cdot}86$	$+\ 8{\cdot}35$	$+18{\cdot}82$	$+\ 2{\cdot}53$	$+37{\cdot}32$	$+47{\cdot}59$	$+11{\cdot}11$	$-\ 1{\cdot}55$	$=+0{\cdot}84$
$+15{\cdot}99$	$+\ 9{\cdot}32$	$-\ 8{\cdot}69$	$-\ 6{\cdot}64$	$+14{\cdot}95$	$+11{\cdot}11$	$+48{\cdot}42$	$+32{\cdot}85$	$=+5{\cdot}65$
$+16{\cdot}84$	$+\ 8{\cdot}72$	$-17{\cdot}64$	$-18{\cdot}83$	$+\ 0{\cdot}19$	$-\ 1{\cdot}55$	$+32{\cdot}85$	$+58{\cdot}26$	$=+4{\cdot}09$

Eliminating from these equations the values of $\Delta\beta$ and z, with their weights, we get :—

$$\Delta\beta = -0{\cdot}0012^{n}. \quad \text{Weight } 4.$$

$$z = +0{\cdot}1091. \quad \text{,, } 22.$$

From Solution I., $z=+0^{R}{\cdot}1092$, which differs only $-0^{R}{\cdot}0001$ from the above solution, so that the difference would in no case influence the residuals in the third decimal place. The value of $\Delta\beta$ is so small that it would change the residuals only $0^{R}{\cdot}002$ in a few of the equations of the group in $x_{,,,,}$ and would change the majority of the residuals by $0^{R}{\cdot}001$ or less. The values of the other unknown quantities and the probable error of one observation will therefore remain sensibly the same as in the original solution.

Remembering that $1^{R}=12''{\cdot}865$, and $\pi=\dfrac{z}{1{\cdot}881}$, we have—

$$\Delta\beta = -0''{\cdot}015 \qquad \text{Prob. error } \pm 0''{\cdot}059$$

$$\pi = +0{\cdot}746 \qquad \text{,, ,, } \pm 0{\cdot}013$$

whilst the original solution gave the almost identical value

$$\pi = +0''{\cdot}747 \qquad \text{Prob. error } \pm 0''{\cdot}013.$$

Thus we find that the true value of the term $\Delta\beta$ (whose factor is $\cos(p-q)\tan\zeta$) is exceedingly small, and is determined, as might be expected, with small weight, its probable error being nearly four times greater than itself, and *its most probable value*

has the opposite sign from that derived for it by Professor Rambaut.

The important significance of this solution is—

1. That the corrections x_I, x_{II}, x_{III}, and x_{IIII}, introduced into the original equations, represent true subjective errors which depend upon the direction of measurement with reference to the line joining the observer's eyes, and which, within the limits of their respective groups, are independent of the zenith distance or the inclination of the line of measurement to the vertical.

2. That, within limits determinable by this series of observations, the influence of atmospheric chromatic dispersion on the measures is insensible.

3. That the value of π (0″·780) found by Professor Rambaut is utterly at variance with the observations, and that the true value of the parallax of α *Centauri* relative to the comparison stars α and β cannot sensibly differ from 0″·74 or 0″·75.

Parallax of β Orionis.

The following observations were made by me for the determination of the parallax of *β Orionis*. The details of these and other observations connected with determination of stellar parallax will appear in vol. viii. of the *Annals of the Cape Observatory*; but as this particular series bears upon and illustrates the subject of the present paper, the results are here given and discussed.

The comparison stars employed were

α = D.M. − 9°·997 Mag. 8·5
β = D.M. − 8·1078 „ 8·4

And their position angles and distances from *β Orionis* are, approximately, for 1890·0—

β Orionis and α	$p = 297°{\cdot}25$	$s = 2960''$
β	118·81	3283

The observations were made with the 7-inch Repsold Heliometer. The image of *β Orionis* was reduced by screens to near equality of magnitude with the comparison stars. A reversing prism was employed, which in 1888 and 1889 was turned 90° after each pointing, and in 1890 after each second pointing. The observations were arranged in the order α, β, β, α, or β, α, α, β, so as to be strictly symmetrical. In the observations of 1888 and 1889 each observation consisted of four pointings (two in each of the two positions of the segments), so that each value of the distances α and β in the following table is the mean of eight pointings.

In the observation of 1890 each observation consisted of eight pointings (four in each position of the segments), so that each measured distance α or β in the following table depends on sixteen pointings; but the pointings in 1890 were more rapidly made than those of 1888 and 1889.

Three observations, made under very bad circumstances of

definition, are rejected ; the others here employed were made under favourable conditions.

The observations divide themselves into five groups, viz.—

A and C made near epochs of parallax maximum at western hour angles.

B made near an epoch of parallax minimum, eastern hour angles.

D made near an epoch of zero parallax, eastern hour angles.

E made near the same epoch as D, but in western hour angles.

The readings are converted into arc with mean scale value and corrected for refraction and aberration.

The right-hand column gives the value of $\beta - \alpha$ corrected for changes in the scale value by the formula

$$\Delta(\beta-\alpha)=\frac{\beta_0-\alpha_0}{\beta_0+\alpha_0}\left[(\beta_0+\alpha_0)-(\beta+\alpha)\right],$$

where β_0 and α_0 represent the mean values of the distances β and α as derived from all the observations.

1888.	α.	β.	$\beta+\alpha$.	$\beta-\alpha$ $+\Delta(\beta-\alpha)$.	Group.
March 17	2959″·848	3283″·309	6243″·16	323″·437	
22	·543	·040	2·58	·500	
27	·680	·127	2·81	·447	A
29	·569	·023	2·59	·456	
1888.					
Sept. 28	2959·529	3283·011	6242·54	323·487	
29	·649	·025	·67	·375	
Oct. 1	·592	·170	·76	·572	B
2	·553	·114	·67	·560	
5	·534	·093	·63	·559	
1889.					
March 6	2959·539	3283·166	6242·71	323·623	
16	·462	·014	·48	·560	
21	·720	·043	·76	·317	C
22	·593	·165	·76	·566	
1890.					
Dec. 20	2959·369	3282·981	6242·35	323·627	
21	·436	2·991	·43	·566	
27	·472	3·190	·66	·716	D
30	·552	3·081	·63	·530	
31	·485	2·977	·46	·501	
1890.					
Dec. 22	2959·542	3283·048	6242·59	323·509	
30	·567	3·060	·63	·494	
31	·509	·187	·70	·680	E
1891.					
Jan. 1	·480	·108	·59	·630	
2	·512	·051	·56	·543	

F

The parallax factors for measures of distance are—

β *Orionis* from star α ... $0\cdot833\ \mathrm{R}\ .\ \cos(\odot - 184^\circ\cdot7)$

,, β ... $0\cdot873\ \mathrm{R}\ .\cos(\odot - 5^\circ\cdot1)$.

The sum and difference of these factors are—

$\beta - \alpha$ ... $1\cdot756\ \mathrm{R}\ .\ \cos(\odot - 4^\circ\cdot5)$

$\beta + \alpha$ ... $0\cdot019\ \mathrm{R}\ .\ \cos(\odot - 128^\circ\cdot5)$.

The form of the equations is—

$$x + (t-1890)\,y + \left\{\begin{matrix}\tan\zeta_\beta\,.\cos(p_\beta - q_\beta)\\ -\tan_\alpha\,.\cos(p_\alpha - q_\alpha)\end{matrix}\right\}\Delta\beta + 1\cdot756\ \mathrm{R}\ .\ \cos(\odot - 4^\circ\cdot5)\,\pi = n$$

where t is the epoch of observation, p_β and p_α the position angles of the stars β and α from β *Orionis*, q_β and q_α the parallactic angle of β *Orionis* at the epochs of observation of the distances β and α respectively, and

$$n = \beta - \alpha + \Delta\,(\beta - \alpha)\ \ldots\ -323''\cdot540.$$

The resulting equations are :—

				Residuals (O−C). I.	II.	III.	IV.
$x - 1\cdot79y$	$+2\cdot59\Delta\beta$	$+1\cdot74\pi$	$= -0''\cdot103$	$-0''\cdot020$	$-0''\cdot031$	$-0''\cdot021$	$-0''\cdot039$
$x - 1\cdot78$	$+1\cdot19$	$+1\cdot75$	$= -\ 040$	$+\ \cdot014$	$+\ \cdot031$	$+\ \cdot024$	$+\ \cdot024$
$x - 1\cdot76$	$+1\cdot90$	$+1\cdot75$	$= -\ \cdot093$	$-\ \cdot026$	$-\ \cdot021$	$-\ \cdot021$	$-\ \cdot030$
$x - 1\cdot76$	$+2\cdot57$	$+1\cdot75$	$= -\ \cdot084$	$-\ \cdot004$	$-\ \cdot013$	$+\ \cdot009$	$-\ \cdot021$
$x - 1\cdot26$	$-0\cdot81$	$-1\cdot76$	$= -\ \cdot053$	$-\ \cdot025$	$-\ \cdot023$	$-\ \cdot031$	$-\ \cdot009$
$x - 1\cdot25$	$-0\cdot38$	$-1\cdot76$	$= -\ \cdot165$	$-\ \cdot129$	$-\ \cdot135$	$-\ \cdot138$	$-\ \cdot121$
$x - 1\cdot25$	$-0\cdot61$	$-1\cdot75$	$= +\ 032$	$+\ \cdot064$	$+\ \cdot062$	$+\ \cdot055$	$+\ \cdot076$
$x - 1\cdot25$	$-0\cdot63$	$-1\cdot75$	$= +\ \cdot020$	$+\ \cdot051$	$+\ \cdot050$	$+\ \cdot043$	$+\ \cdot064$
$x - 1\cdot24$	$-0\cdot63$	$-1\cdot74$	$= +\ \cdot019$	$+\ \cdot048$	$+\ \cdot049$	$+\ \cdot042$	$+\ \cdot062$
$x - 0\cdot82$	$+1\cdot59$	$+1\cdot65$	$= +\ \cdot083$	$+\ \cdot113$	$+\ \cdot119$	$+\ \cdot119$	$+\ \cdot120$
$x - 0\cdot80$	$+1\cdot96$	$+1\cdot73$	$= +\ \cdot020$	$+\ \cdot057$	$+\ \cdot056$	$+\ \cdot060$	$+\ \cdot047$
$x - 0\cdot78$	$+2\cdot01$	$+1\cdot75$	$= -\ \cdot223$	$-\ \cdot187$	$-\ \cdot188$	$-\ \cdot183$	$-\ \cdot197$
$x - 0\cdot78$	$+2\cdot27$	$+1\cdot75$	$= +\ \cdot026$	$+\ \cdot067$	$+\ \cdot061$	$+\ \cdot069$	$+\ \cdot052$
$x + 0\cdot97$	$-0\cdot63$	$-0\cdot18$	$= +\ \cdot087$	$+\ \cdot030$	$+\ \cdot046$	$+\ \cdot035$	$+\ \cdot047$
$x + 0\cdot97$	$-0\cdot64$	$-0\cdot15$	$= +\ \cdot026$	$-\ \cdot032$	$-\ \cdot015$	$-\ \cdot026$	$-\ 014$
$x + 0\cdot99$	$-0\cdot68$	$+0\cdot03$	$= +\ \cdot176$	$+\ \cdot115$	$+\ \cdot135$	$+\ \cdot122$	$+\ \cdot136$
$x + 0\cdot99$	$-0\cdot40$	$+0\cdot10$	$= -\ \cdot010$	$-\ \cdot066$	$-\ \cdot050$	$-\ \cdot060$	$-\ \cdot050$
$x + 1\cdot00$	$-0\cdot36$	$+0\cdot16$	$= -\ \cdot039$	$-\ \cdot094$	$-\ \cdot079$	$-\ \cdot089$	$-\ \cdot080$
$x + 0\cdot97$	$+1\cdot19$	$-0\cdot02$	$= -\ \cdot031$	$-\ \cdot051$	$-\ \cdot071$	$-\ \cdot061$	$-\ \cdot071$
$x + 0\cdot99$	$+1\cdot22$	$+0\cdot10$	$= -\ \cdot046$	$-\ \cdot070$	$-\ \cdot086$	$-\ \cdot076$	$-\ \cdot086$
$x + 1\cdot00$	$+1\cdot33$	$+0\ 16$	$= +\ \cdot140$	$+\ \cdot118$	$-\ \cdot100$	$+\ \cdot111$	$+\ \cdot099$
$x + 1\cdot00$	$+1\cdot29$	$+0\cdot19$	$= +\ \cdot090$	$+\ \cdot066$	$+\ \cdot050$	$+\ \cdot060$	$+\ \cdot049$
$x + 1\cdot00$	$+1\cdot37$	$+0\cdot22$	$= +\ \cdot003$	$-\ \cdot020$	$-\ \cdot037$	$-\ \cdot026$	$-\ \cdot038$

Giving weight $1\frac{1}{2}$ to the last two groups of equations, which depend on double the number of pointings each, and weight 1 to the rest, the normal equations are—

$+28{\cdot}0x$	$-\ 1{\cdot}7y$	$+18{\cdot}6\Delta\beta$	$+\ 5{\cdot}9\pi$	$=+0''{\cdot}029$
$-\ 1{\cdot}7$	$+37{\cdot}6$	$-11{\cdot}4$	$-\ 6{\cdot}1$	$=+1{\cdot}407$
$+18{\cdot}6$	$-11{\cdot}4$	$+50{\cdot}6$	$+34{\cdot}6$	$=-0{\cdot}807$
$+\ 5{\cdot}9$	$-\ 6{\cdot}1$	$+34{\cdot}5$	$+39{\cdot}7$	$=-0{\cdot}447$

Of these normals the following solutions have been made :—

I. Solved for all terms.
II. Rejecting the normal and terms in $\Delta\beta$.
III. Rejecting the normal and terms in π.
IV. Rejecting the normal and terms in $\Delta\beta$ and π.

These solutions give the following values and weights of the unknown quantities :—

Solution.	x.	Weight.	y.	Weight.	$\Delta\beta$.	Weight.	π.	Weight.
I.	$+0''{\cdot}014$	18·0	$+0''{\cdot}033$	34·2	$-0''{\cdot}020$	12·9	$+0''{\cdot}009$	13·7
II.	+ ·004	27·1	+ ·036	36·6	...		− ·006	37·6
III.	+ ·011	21·0	+ ·034	34·7	− ·012	35·4	...	
IV.	+ ·003	27·9	+ ·037	37·5	...		...	

The substitutions of these values in the original equations give the residuals contained in the columns to the right-hand side of the equations. The corresponding values of $[p\,.\,vv]$, $\frac{[p\,.\,vv]}{n-m}$, the probable error of a single observation of weight unity, and the probable errors of the unknown quantities are given in the following table for each solution :—

Solution.	$[p\,.\,vv]$	$\frac{[p\,.\,vv]}{n-m}$	Prob. error of 1 Obs. of Weight 1.	x.	Probable errors of y.	$\Delta\beta$.	π.
I.	0·165050	0·008687	$\pm 0''{\cdot}0621$	$\pm 0''{\cdot}0147$	$\pm 0''{\cdot}0106$	$\pm 0''{\cdot}0173$	$\pm 0''{\cdot}0168$
II.	·169813	·008490	·0614	·0118	·0103	...	·0100
III.	·165791	·008290	·0607	·0133	·0103	·0102	...
IV.	·172825	·008230	·0605	·0115	·0099	...	...

Thus Solution IV. gives the best representation of the observations, proving conclusively that neither $\Delta\beta$ nor π have sensible values.

(We have here also a practical example of the fallacy already referred to, viz. that the representation of equations is necessarily improved by the employment of additional unknown quantities.)

The conclusion is that *β Orionis* probably belongs to a great group or system of very distant stars, to which group or system

the comparison stars α and β also belong, and the relative annual parallaxes of the stars of this group probably do not exceed $0''{\cdot}01$.

The mean light of β *Orionis* is also probably of the same mean refrangibility as that of the comparison stars.

In connection with both these conclusions it will be interesting to determine whether the comparison stars α and β have spectra like β *Orionis*, with the same characteristic hydrogen and helium lines.

On the somewhat improbable assumption that the value of $\Delta\beta$, as determined from Solution III., is a true quantity, and also that the observer in his heliometer pointings brings into coincidence the *brightest* parts of the dispersion spectra and not their *similarly coloured* parts, we may determine the difference in wave-length between the mean light of β *Orionis* and that of the mean light of the comparison stars.

If μ_0 is the refractive index of air of the mean wave-length for which we have computed our mean refractions, and $\partial\mu$ is the difference between the refractive index of air for light of the mean wave-length of β *Orionis* and that of the comparison stars, then

$$\partial\mu = \frac{\Delta\beta(\mu_0 - 1)}{\beta_0}$$

We have thus

$\Delta\beta = -0''{\cdot}012 \pm 0''{\cdot}010$ from Solution III.

$\mu_0 - 1 = 0{\cdot}00029508$, $\beta_0 = 57''{\cdot}652$ } as adopted by Rambaut (*loc. cit.*) (apparently the mean of Kettler's values for D and E).

Hence

$$\partial\mu = -0{\cdot}00000006 \pm 0{\cdot}00000005,$$

which corresponds with about one 19th part of the difference of μ for light of wave-lengths D and E.

On a method of Determining the effect of Atmospheric Dispersion on the value of the Solar Parallax.

I have a strong impression that when heliometer observations are made at sufficient zenith distance for atmospheric dispersion to produce spectra of sensible length, the observer's tendency is to bring the similarly coloured parts of the spectra rather than their brightest parts to coincidence.

In observations with the filar micrometer, on the other hand, it is probable that the tendency of the observer will be to place the dark web on the brightest part of the spectrum, irrespective of colour, as Rambaut's observations of β *Cygni* and γ *Andromedæ* appear to show.

Some of my heliometer observations of *Mars* in 1877 were

made at zenith distances of 70° and upwards, and I have still a distinct recollection, in such circumstances, of the marked prismatic colouration both of the limbs of the planet and of the star discs (the length of the visible spectrum must have been then over 2″·5). I believe that in such measures I matched the colours of the spectra of the star and limb of the planet; nor do I believe that pointings which would satisfy the eye of the observer could be made in any other way. I also believe that it is on account of this method of pointing that the resulting value of the solar parallax from the Mars observations was not affected with larger systematic error.

In observations of stars, and especially where both stars are reduced by screens to approximate equality of magnitude, I believe the tendency to match the colours may be even more strongly marked.

Except in the case of *α Centauri*, where the near neighbourhood of the companion star seemed likely to produce subjective errors depending on the direction of measurement, I can find in my own observations no proof whatever of any difference between the E. and W. observations such as would be produced by atmospheric dispersion. Thus, in my observations for the parallax of *Sirius*, *ε Indi*, and *Lacaille* 952 (*Mem. R.A.S.* vol. xlviii.), and also in that of *β Orionis* just described, although $\Delta\beta$ has large factors of opposite sign on opposite sides of the meridian, the values of $x_w - x_e$ are always small, and the probable errors of their determination are as large as, and generally much larger than, the quantity itself.

It does not appear necessary to discuss these observations farther, because, with specially devised observations and modern heliometers used with the reversing prism, the question whether the positions of strongly coloured stars can be determined by heliometer observations free from the effects of chromatic dispersion can be much more conclusively settled as follows :—

Let a strongly coloured star whose declination is nearly of the same name as that of the latitude of the observatory be selected, and this star should be preceded and followed by convenient normally coloured comparison stars of nearly the same declination, situated as symmetrically as possible with respect to the coloured star. The distances of these comparison stars from the coloured star, if measured at considerable hour angles both E. and W. of the meridian, will afford large factors of $\Delta\beta$. Thus, at the Cape, suppose a red star, whose declination is $-35°$, has two comparison stars, a and b, whose position angles from it are 270° and 90° respectively, and the distances nearly equal. If these distances a and b are observed like the observations of *β Orionis*—but at 5^h hour angle, both E. and W. of the meridian—we have $\zeta = 60°{\cdot}3$, and $(p-q) = 157°{\cdot}4$ and $337°{\cdot}4$ for the eastern observations, and $22°{\cdot}6$ and $202°\,6$ for the western. The factor of $\Delta\beta$ (viz. $\tan\zeta\cos(p-q)$) will be—

For east observations { Star a Factor $-1\cdot615$; Star b Factor $+1\cdot615$ } Factor for $a-b$ ... $-3\cdot23$

For west observations ... ... $+3\cdot23$

Factor W$-$E $+6\cdot46$

therefore

$$\Delta\beta = \frac{x_W - x_E}{6\cdot46}$$

Thus, if observations are made of the same accuracy as those for the parallax of β *Orionis* given in the present paper, we should have from one observation each E. and W. of the meridian—

$x_W - x_E$ with the probable error $\pm 0''\cdot0605\sqrt{2} = \pm 0''\cdot086$.

and we determine $\Delta\beta$ with the probable error

$$\frac{\pm 0''\cdot086}{6\cdot4} = \pm 0''\cdot013$$

I have accordingly selected the following coloured stars with suitable comparison stars for observation at the Cape, and another series suitable for northern observatories. This selection has, of course, been very much limited by the necessity for finding also symmetrically situated comparison stars in position angles near 90° and 270°.

It is much to be desired that heliometer observers in the northern hemisphere, *and especially those who co-operated in the observations of Iris, Victoria, and Sappho in* 1888 *and* 1889, should co-operate in this programme. From four to six observations each E. and W. should be sufficient for each set of stars. The results of such a series of observations cannot fail to throw much light upon a most fundamental question in practical astronomy. The last uncertainty in our knowledge of the solar parallax rests almost entirely on the answer which these results would give to the question, What is the limiting value of $\Delta\beta$ for the mean light of the average minor planet compared with that of the average comparison star? That value will certainly be smaller than the value of $\Delta\beta$ for the very deeply coloured stars here selected, because the difference in colour between *Victoria* and *Sappho* and their comparison stars was not perceptible when their images were juxtaposed, and, in the case of *Iris*, only Dr. Elkin noted a reddish colour in the planet.

If the results of observation should prove that $x_W - x_E$ is insensible for such highly coloured stars, then it will be certain that in his pointings with the heliometer the observer unconsciously brings the similarly coloured and not the brightest parts of the dispersion spectra into coincidence. On the other hand, if the highly coloured stars show appreciable values of $x_W - x_E$, the question arises, How are the results to be rigorously translated

into the corresponding effect of atmospheric dispersion upon the value of the solar parallax which has been determined from the observations of *Iris*, *Victoria*, and *Sappho*?

I have long had this question under consideration, and have at last found what promises to be a satisfactory solution. Let the photographic magnitudes of the stars in the proposed programme be determined by the same process which Kapteyn has employed for the magnitudes of the C.P.D., and let their visual magnitudes be also carefully determined.

Let p be the photographic magnitude of any star.
v the visual magnitude of the same star.
p_a, p_b, &c., v_a, v_b, &c., the photographic and visual magnitudes of the coloured stars a, b, &c. respectively.

Assume, for the moment, that the comparison stars of the coloured stars are of normal colour; then the proposed observations will, when discussed in the manner already described, give values of $\Delta\beta_a$, $\Delta\beta_b$, &c.

It is evident that

$(p_a - v_a)$ is a measure of what we may call the redness of star a;
$(p_b - v_a)$ is a measure of what we may call the redness of star b, &c.;
and that $\Delta\beta_a$ must be some function of p_a and v_a.
$\Delta\beta_b$ must be some function of p_b and v_b &c.

and if a linear function we should have

$$\frac{\Delta\beta_a}{\Delta\beta_b} = \frac{p_a - v_a}{p_b - v_b}$$

or

$$\Delta\beta_a = \gamma\,(p_a - v_a)$$
$$\Delta\beta_b = \gamma\,(p_b - v_a) \quad \&c.$$

where γ is a constant to be determined.

If necessary, the range of p and v can be varied by the heliometer observation of stars of different magnitudes and different depths of colour, and we may thus determine whether

$$\gamma \text{ is} = \frac{\Delta\beta}{(p-v)} + \frac{\Delta\beta}{(p-v)^2} + \&c.$$

or is some other function of p and v.

But as we know that $\Delta\beta$ is very small, we may for our present purposes assume that γ is a constant.

From the observed values of $\Delta\beta_a$, $\Delta\beta_b$, &c. and of $(p_a - v_a)$, $(p_b - v_b)$, &c. we then derive the mean value of γ.

The visual magnitude of the minor planet would, of course, be determined in the ordinary way. In determining the photographic magnitude of the planet it will be best to make two exposures upon each plate, shifting the reading of the scales of the

eye-end of the guiding telescope between the exposures, employing the planet as a guiding star in one exposure, and a fixed star as the guide in an immediately subsequent exposure of equal length. The photographic magnitude of the planet will then be obtained by comparison of the diameter of its photographic disc with the diameters of a number of star-discs on the same plate which represent stars of known visual magnitude; in fact, by a process exactly similar to that by which the photographic magnitudes of our coloured stars are determined. The results will not be affected by the real diameter of the planet, because that diameter is smaller than the granules of the film. The diameter of the planet's photographic disc will thus be the same function of its photographic magnitude as if the planet were a fixed star.

Let p_p and v_p represent the photographic and visual magnitudes of the planet.

We then have

$$\Delta\beta_p = \gamma\,(p_p - v_p),$$

where γ, p_p, and v_p have been determined.

It then only remains in the discussions of the parallax of *Iris*, *Victoria*, *and Sappho* to introduce the terms $\tan\zeta\cos(p-q)\,\Delta\beta_p$ into the equations resulting from the measures of distance, and $\tan\zeta\sin(p-q)\,\Delta\beta_p$ into those resulting from the observations of position angle, and to find the value of the $\odot^r$ parallax in terms of $\Delta\beta_p$; a matter not of very great labour when the original computations are available.

The factor with which $\Delta\beta_p$ enters into the value of the solar parallax will be, roughly, the mean tangent of the zenith distances at which the observations were made. Say, still more roughly, 1·4 (corresponding to the tangent of 55°); but as the distances of the planets are sensibly less than unity, the factor will probably come out somewhat less than 1·4. Having regard to the very large factors with which $\Delta\beta$ may be determined, and the high accuracy with which the observations can be made, there seems little doubt that, if the proposed observations are earnestly taken up, a completely satisfactory result will be arrived at.

For example. It has already been shown that if the observations for determining $\Delta\beta$ for one of the very red stars, *r*, can be made, under the prescribed conditions, 5^h E. & W. of the meridian *on a single night*, with the same accuracy as the observations of *β Orionis*, we should obtain from this star the value of

$$\Delta\beta_r \text{ with the probable error } \pm 0''{\cdot}013.$$

The difference between the photographic and visual magnitudes of this star may be three magnitudes (see list below), hence

$$p_r - v_r = 3,$$

and thus the probable error of the determination of γ will be

$$\frac{p \,.\, e \text{ of } \Delta\beta_r}{3} = \frac{\pm 0''\cdot 013}{3} = \pm 0''\cdot 0043 .$$

Now it is extremely improbable that the difference between the visual and photographic magnitudes of the planets will be greater than one magnitude ; so we may put

$$p_p - v_p = < 1 .$$

Hence if $\Delta \beta_p$ appears in the value of the solar parallax with the factor 1·4, we shall have

The probable error of the determination of the solar parallax, so far as it depends on the uncertainty of the effect of atmospheric dispersion $\Big\}$ $< \pm 0''\cdot 0043 \times 1\cdot 4 = < \pm 0''\cdot 006 .$

Of course, this result does not include the effect of the probable error of the determination of $(p_r - v_r)$ and $(p_p - v_p)$, but since these cannot exceed 0·2 or 0·3 magnitudes, and will probably be less, they must have a very small influence on the result.

At considerable zenith distances the probable error of $x_w - x_g$ will also probably be greater than that found from the observations of *β Orionis* ; but, even if that error is two or three times as great, an amply sufficient accuracy will be secured by making four or five observations, each E. & W. of four or five different sets of stars, as in the programme of observation.

It is possible that if the observers are influenced in their habit of pointing partially by the points of similar colour in the short spectra under observation and partially by the points of maximum light, the value of γ, and consequently of $\Delta \beta_p$, may be different for different observers. In this case it would only be necessary to introduce $\Delta \beta'_p$, $\Delta \beta''_p$, &c., into the corresponding equations resulting from the observations of the different observers. But, from the fact that precisely the same value of the solar parallax was obtained from the diurnal parallax of *Victoria* at the Cape alone as that derived from the combination of the Cape observations with those of the northern observatories, it seems probable that such differences, if they exist, must be very small indeed.

The sets of stars proposed for the heliometer-observation are the following :—

The photographic magnitudes for the southern stars are provisionally taken from the C.P.D. The northern stars have been selected from Krüger's *Catalog der farbigen Sterne.* The photographic magnitudes of these northern stars cannot be satisfactorily determined at the Cape, and northern observers are earnestly requested to take the necessary plates. In order that these photographic

measures of redness may be strictly comparable, it is desirable that all observers should employ the same kind of plate. The Ilford "special rapid plates," with an exposure of twenty minutes, are suggested. If such plates are sent to the Cape they will be measured and discussed there.

It is desirable to photograph not merely the red star, but to make three separate plates for each group—one having the preceding star in the centre of the plate, one having the red star in the centre of the plate, one having the following star in the centre of the plate. On each plate there may be two or three exposures separated by 1^{mm}.

Southern Heliometer Programme.

Group. Culmination at Midnight.	Magnitude. Visual.	Photo.		C.P.D. 1875.		Distance.	Position Angle.	α_0. 1900.	δ_0.
				h m s	° ′	′	°	h m s	° ′
I. Jan. 13	7·0	6·4		7 40 7·2	−37 38·5	8·2	279·8	7 41 20	−37 42
	3·5	6·8	Orange	40 48·7	39·9	11·1	78·5	7 42 19	−37 42
	6·5	6·4		41 43·3	37·7				
II. Feb. 20	7·5	8·0		10 10 53·5	−49 33·2	16·6	252·8	10 21 41	−49 38
	8·0	Not in C.P.D.*	V. red	12 31·4	28·4	17·7	93·9	10 14 25	−49 36
	8·5	8·3		14 20·1	29·6				
III. Apr. 21	8·3	8·8		13 51 41·9	−26 3·6	101·7	270·6	13 59 53	−26 11
	3·6	5·3	Deep orange	13 59 15·9	4·8	91·8	88·0	14 4 5	−26 10
	6·8	7·1		14 6 5·5	1·6				
IV. June 15	7·7	8·0		17 32 13·4	−35 34·5	83·8	273·1	17 37 20	−35 38
	8·0	9·6	Very red	39 4·1	39·0	71·0	87·0	17 43 30	−35 38
	7·1	7·6		44 33·4	35·3				
V. June 24	6·8	8·2		18 7 4·9	−29 51·4	76·9	271·0	18 11 38	−29 51
	2·8	6·2	Reddish orange	12 59·8	52·7	45·8	87·3	16 21	−29 51
	8·4	8·2		16 30·8	50·5				
VI. July 22	8·0	8·0		20 5 53·9	−21 48·8	54·7	262·8	20 9 19	−21 41
	7·7	10·4	V. v. red	9 47·0	42·1	62·6	96·0	13 29	−21 41
	7·6	7·6		14 15·1	48·5				

* The star is certainly photographically fainter than $9^m·6$, not being visible on any of the C.P.D. plates, which contain stars of that photographic magnitude. Mag. in Z.C. 8·2, in C.G.A. 7·8; Cape 13/2 and 15/3/97. Redness = 8, mag. 8·0. Gould calls the star crimson.

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Northern Heliometer Programme.

Group. Culmination at Midnight.	Magnitude. Visual.	Photo.		1875·0.		Distance.	Position Angle.	α_0. 1900·0.	δ_0.
				h m s	° ′	′	°	h m s	° ′
VII. Nov. 27	6·5	...	Lal. 4549	3 58 43·0	+32 3·0	80·3	263·0	4 3 27	+32 11
	6·5	...	Kr. 328, O.R.	4 5 0·7	12·6				
	7·4	...	B.D.+32°, 765	10 14·9	5·2	66·0	96·5	4 9 13	+32 12
VIII. Dec. 9	8·1	...	B.D.+43°, 1164	4 52 38·4	+43 14·8	115·8	268·8	4 59 44	+43 18
	8·3	...	Kr. 447, R.	5 3 14·3	17·2				
	8·4	...	B.D.+43, 1250	5 11 57·4	16·9	95·2	90·2	5 9 23	+43 19
IX. Dec. 11	6·7	...	Paris 6038	5 8 2·9	+34 10·0	56·2	271·8	5 11 57	+34 11
	7·9	...	Kr. 461, O.R.'	12 34·2	8·2				
	7·3	...	Paris 6222	17 14·3	4·2	58·1	93·9	5 16 33	+34 8
X. Dec. 28	8·3	...	W.B. 497	6 19 39·4	+38 22·5	97·9	264·5	6 25 31	+38 26
	7·0	...	Kr. 604, R.R.	27 57·1	32·6				
	7·5	...	Paris 8097	36 53·3	30·5	104·8	91·9	6 34 8	+38 30